A thermodynamical approach to contact wear as application of moving discontinuities

Claude Stolz

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Abstract The propagation of moving surface inside a body is analysed within the framework of thermomechanical couplings, when the moving surface is associated with an irreversible change of mechanical properties. The moving surface is a surface of heat sources and of entropy production, intensities of which are related to particular energy release rates defined in terms of Hamiltonian gradients. As example, we analyse wear process. Wear phenomena due to contact and relative motion between two solids depend on the loading conditions and material mechanical properties. Friction between contacting bodies induces damage of materials, producing surface and subsurface cracks. Particles are detached from sound solids when some local criteria are satisfied at the boundary. By the way as wears occurs, geometrical changes take place and contact conditions are modified, the particle induce a specific layer with particular properties. Then the interface between the bodies is a complex medium made of detached particles, eventually a lubricant fluid, and damaged zones.We propose to describe the evolution of the interface using the framework developped before for inducing the general form of a wear-law.

1 Introduction

In the recent past, the propagation of damage has been studied in connection with fracture mechanics and different approaches based on macroscopic or microscopic descriptions of mechanical degradation properties have been proposed.

During a loading history damage in continuum mechanics can be induced by the initiation and the growth of micro-cracks and micro-cavities. These descriptions, which are based on the evolution of the microscopic properties, propose to take the growth of pores or micro-cracks into account, through the idea that when some threshold value is reached, the material cannot support further tensile loading.

Variational formulations were performed to describe the evolution of the surface between the sound and damaged material [3][9]. In the framework of thermomechanical coupling as in fracture mechanics the analysis defines two different energy release rates associated with heat production and entropy production. [10].

This paper is concerned mostly with the description of damage involved on the evolution of a moving interface along which mechanical transformation occurs. Some connections can be made with the notion of configurational forces, [6][8][11][5].

C. Stolz
CNRS UMR7649, Laboratoire de Mécanique des Solides,
Ecole Polytechnique, 91128 Palaiseau France
E-mail: stolz@lms.polytechnique.fr

2 General features.

The domain Ω is composed of two distinct volumes Ω_1, Ω_2 of two materials with different mechanical characteristics. The bounding between the two phases is perfect and the interface is denoted by Γ, ($\Gamma = \partial\Omega_1 \cap \partial\Omega_2$). The external surface $\partial\Omega$ is decomposed in two parts $\partial\Omega_{\rm u}$ and $\partial\Omega_{\rm T}$ on which the displacement $\mathbf{u}^d$ and the loading $\mathbf{T}^d$ are prescribed respectively.

The material 1 changes into material 2 along the interface Γ by an irreversible process. Hence Γ moves with the normal velocity $\mathbf{c} = \phi\boldsymbol{\nu}$ in the reference state, $\boldsymbol{\nu}$ is the outward Ω_2 normal, then ϕ is positive.

When the surface Γ is moving, all the mechanical quantities f can have a jump denoted by $[\![f]\!]_\Gamma = f_1 - f_2$, and any volume average has a rate defined by

$$\frac{\rm d}{{\rm d}t}\int_{\Omega(\Gamma)} f \,{\rm d}\Omega = \int_{\Omega(\Gamma)} \dot{f} \,{\rm d}\Omega - \int_\Gamma [\![f]\!]_\Gamma \; c.\boldsymbol{\nu} \,{\rm d}S \tag{1}$$

The state of the system is characterized by the displacement field $\mathbf{u}$, from which the strain field ε is derived. The other parameters are the temperature T and the spatial distribution of the two phases given by the position of the boundary Γ.

We analyse quasistatic evolution of Γ under given loading and displacements prescribed on the boundary $\partial\Omega$.

The behaviour of the phase i is defined by the free energy density w_i, function of the strain ε and of the temperature T. The mass density ρ_i of each phase is the same ($\rho = \rho_i$). The state equations of each phase are

$$\sigma_i = \rho\frac{\partial w_i}{\partial\varepsilon}, \quad s_i = -\frac{\partial w_i}{\partial T}, \tag{2}$$

where σ_i is the reversible stress and s_i the entropy. If the materials have no viscosity then σ_i is the stress satisfying the momentum equation.

Assume now that the two phases are linear elastic materials and that the temperature is fixed.

The two phases are linear elastic. The potential energy $\mathcal{E}$ of the structure Ω ($\Omega_1 \cup \Omega_2$) has the following form

$$\mathcal{E}(\mathbf{u}, T, \Gamma, \mathbf{T}^d) = \sum_{i=1,2}\int_{\Omega_i} \rho\; w_i(\varepsilon(u), T) \,{\rm d}\Omega - \int_{\partial\Omega_{\rm T}} \mathbf{T}^d.\mathbf{u} \,{\rm d}S,$$

where $\partial\Omega_{\rm T} \cup \partial\Omega_{\rm u} = \partial\Omega$. The potential energy plays the role of the global free energy in a thermodynamical description ; we can notice that the position of the interface Γ becomes an internal parameter for the global system. The characterization of an equilibrium state is given by the stationarity of the potential energy

$$\frac{\partial\mathcal{E}}{\partial\mathbf{u}}\cdot\delta\mathbf{u} = \sum_{i=1,2}\int_{\Omega_i} \rho\frac{\partial w_i}{\partial\varepsilon} : \varepsilon(\delta\mathbf{u}) \,{\rm d}\Omega - \int_{\partial\Omega_{\rm T}} \mathbf{T}^d.\delta\mathbf{u} \,{\rm d}S = 0, \tag{3}$$

for all $\delta\mathbf{u}$ kinematically admissible field satsfying $\delta\mathbf{u} = 0$ over $\partial\Omega_{\rm u}$. This formulation is equivalent to the set of local equations :

– local constitutive relations:

$$\sigma_i = \rho\frac{\partial w_i}{\partial\varepsilon} = \mathbb{C}_i : \varepsilon, \text{ on } \Omega_i, \tag{4}$$

– momentum equations

$$\text{div } \sigma_i = 0, \text{ on } \Omega_i, \; [\![\sigma]\!]_\Gamma.\boldsymbol{\nu} = 0 \text{ over } \Gamma, \; \sigma.\mathbf{n} = \mathbf{T}^d \text{ over } \partial\Omega_{\rm T}, \tag{5}$$

– compatibility relations

$$2\varepsilon = \nabla\mathbf{u} + \nabla^t\mathbf{u}, [\![\mathbf{u}]\!]_\Gamma = 0 \text{ over } \Gamma, \; \mathbf{u} = \mathbf{u}^d \text{ over } \partial\Omega_{\rm u}. \tag{6}$$

They are equations of a problem of heterogeneous elasticity. The solution is denoted by $\mathbf{u}^{sol}$, this field depends upon the quantities $(u^d, \mathbf{T}^d, \Gamma)$. For an equilibrium state

$$\mathcal{E}(\mathbf{u}^{sol}, \mathbf{T}^d, \Gamma) = W(\mathbf{u}^d, \mathbf{T}^d, \Gamma). \tag{7}$$

This equation expresses the fact that the position of the interface Γ plays the role of internal parameters.

At a given state of equilibrium for a given value of the prescribed loading ($\mathbf{u}^d, \mathbf{T}^d$), the position for the interface Γ is known. At this time a variation of the loading is imposed, the mechanical quantities evolve and propagation of the interface can occur according to a given evolution law. For a prescribed history of the loading, we must determine the rate of all mechanical fields and the normal propagation ϕ to characterize the position of the interface Γ at each time. Along interface Γ perfect bounding is preserved at each time. Let us introduce the notion of convected derivative.

Convected Derivation : *The convected derivative $\mathcal{D}_\phi$ of any function $f(\mathbf{X}_\Gamma, t)$ is*

$$\mathcal{D}_\phi f = \lim_{\tau \to 0} \frac{f(\mathbf{X}_\Gamma + \phi \boldsymbol{\nu}\tau, t+\tau) - f(\mathbf{X}_\Gamma, t)}{\tau}. \tag{8}$$

With this definition, we can express the transport of the normal vector at point $\mathbf{x}$ as

$$\mathcal{D}_\phi \boldsymbol{\nu} = -\nabla\phi.\mathbf{e}_\alpha \, \mathbf{e}_\alpha, \tag{9}$$

where $\mathbf{e}_1, \mathbf{e}_2$ is a basis of the plane tangent to Γ at point $\mathbf{x}$. We can notice that the equation of the surface Γ, $S(\mathbf{X}, t) = 0$ satisfies immediatedly

$$\mathcal{D}_\phi S = \frac{\partial S}{\partial \mathbf{X}}.\dot{\mathbf{X}} + \frac{\partial S}{\partial t} = 0, \tag{10}$$

which defines the normal velocity $\mathbf{c}$ of Γ :

$$\mathbf{c} = \phi\boldsymbol{\nu}, \quad \boldsymbol{\nu} = \frac{\partial S}{\partial \mathbf{X}} / \parallel \frac{\partial S}{\partial \mathbf{X}} \parallel, \tag{11}$$

and finally for any differentiable fields f the convected derivative is obvious by

$$\mathcal{D}_\phi f = \frac{\partial f}{\partial t} + \phi \, \nabla f.\boldsymbol{\nu} \, . \tag{12}$$

Hadamard's relations The bounding being perfect between the phases, the temperature, the displacement and the stress vector are continuous along Γ. Their rates have discontinuities according to the general compatibility conditions of Hadamard, rewritten in terms of the convected derivative :

– continuity of displacement

$$[\,\mathbf{u}\,]_\Gamma = 0 \Rightarrow \mathcal{D}_\phi([\,\mathbf{u}\,]_\Gamma) = [\,\mathbf{v}\,]_\Gamma + \phi[\,\nabla\mathbf{u}\,]_\Gamma.\boldsymbol{\nu} = 0, \tag{13}$$

– continuity of the stress vector

$$[\,\sigma\,]_\Gamma.\boldsymbol{\nu} = 0 \Rightarrow \mathcal{D}_\phi([\,\sigma\,]_\Gamma.\boldsymbol{\nu}) = [\,\dot{\sigma}\,]_\Gamma.\boldsymbol{\nu} - \mathrm{div}_\Gamma([\,\sigma\,]_\Gamma \, \phi) = 0. \tag{14}$$

The last equation is obtained taking the equilibrium equation into account. As we have

$$\mathcal{D}_\phi([\,\sigma\,]_\Gamma.\boldsymbol{\nu}) = \mathcal{D}_\phi[\,\sigma\,]_\Gamma.\boldsymbol{\nu} + [\,\sigma\,]_\Gamma.\mathcal{D}_\phi\boldsymbol{\nu}, \tag{15}$$

where

$$\mathcal{D}_\phi[\,\sigma\,]_\Gamma = [\,\dot{\sigma}\,]_\Gamma + \phi\boldsymbol{\nu}.[\,\nabla\sigma\,]_\Gamma.\boldsymbol{\nu}, \tag{16}$$

and using the conservation of momentum

$$\mathbf{e}_\alpha.\nabla\sigma.\mathbf{e}_\alpha + \boldsymbol{\nu}.\nabla\sigma.\boldsymbol{\nu} = 0, \tag{17}$$

and the expression of the surface divergence given by

$$\mathrm{div}_\Gamma \, F = \mathrm{div}\, F - \boldsymbol{\nu}.\nabla F.\boldsymbol{\nu}, \tag{18}$$

the above result is obtained.

Orthogonality property for discontinuities. Since the displacement is continuous along the interface,

$$[\,\mathbf{u}\,]_{\Gamma} = 0, \Rightarrow [\,\nabla\mathbf{u}\,]_{\Gamma}.\mathbf{e}_{\alpha} = 0, \tag{19}$$

the discontinuities of the gradient must satisfy

$$[\,\nabla\mathbf{u}\,]_{\Gamma} = \mathbf{U}(x) \otimes \boldsymbol{\nu}. \tag{20}$$

Since the stress vector is continuous on Γ,

$$[\,\sigma\,]_{\Gamma}.\boldsymbol{\nu} = 0, \tag{21}$$

the discontinuities of σ and of $\nabla\mathbf{u}$ have the property of orthogonality as pointed in [7] :

$$[\,\sigma\,]_{\Gamma} : [\,\nabla\mathbf{u}\,]_{\Gamma} = 0. \tag{22}$$

3 Dissipation analysis

The mass conservation leads to the continuity of the mass flux $m = \rho\phi$. The first and the second law of thermodynamics give rise to local equations inside the volume and along the moving boundary Γ :

$$\rho\dot{\mathrm{e}}_i = \sigma_i : \dot{\varepsilon} - \operatorname{div}\mathbf{q}, \text{ over } \Omega_i, \tag{23}$$

$$0 = m[\,\mathrm{e}\,]_{\Gamma} - \boldsymbol{\nu}.\sigma.[\,\mathbf{v}\,]_{\Gamma} + \boldsymbol{\nu}.[\,\mathbf{q}\,]_{\Gamma}, \text{ on } \Gamma, \tag{24}$$

e_i is the internal energy density ($\mathrm{e}_i = w_i + T\ s_i$), and $\mathbf{q}$ is the heat flux associated to the heat conduction.

Thanks to Hadamard compatibility equations, the heat power supply is given in terms of a release rate of internal energy $\mathcal{G}_{th}$ as an objective quantity defined along Γ

$$-\boldsymbol{\nu}.[\,\mathbf{q}\,]_{\Gamma} = \mathcal{G}_{th}\phi, \text{ with } \mathcal{G}_{th} = \rho[\,\mathrm{e}\,]_{\Gamma} - \sigma : [\,\varepsilon\,]_{\Gamma}. \tag{25}$$

The value of $\mathcal{G}_{th}$ is obtained considering the orthogonality condition on the discontinuities. When $\phi = 0$ in the reference state the interface Γ does not move, and the normal heat flux is continuous. When the transformation occurs, the moving interface is a surface of heat sources intensities which are given by $\mathcal{G}_{th}\phi$.

The total internal energy of the structure is

$$\mathrm{E}(\mathbf{u}, \Gamma, T, T^d) = \int_{\Omega(\Gamma)} \rho\ \mathrm{e}\ \mathrm{d}\Omega - \int_{\partial\Omega_{\mathrm{T}}} \mathbf{T}^d.\mathbf{u}\ \mathrm{d}S = \mathcal{E} + \int_{\Omega(\Gamma)} \rho\ sT\ \mathrm{d}\Omega. \tag{26}$$

For quasistatic evolution, the first law of thermodynamics is written as follows :

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{E} - \frac{\partial \mathrm{E}}{\partial \mathbf{T}^d}.\dot{\mathbf{T}}^d = -\int_{\partial\Omega} \mathbf{q}.\mathbf{n}\ \mathrm{d}S, \tag{27}$$

and taking into account the conservation of momentum, we have

$$\frac{\partial \mathrm{E}}{\partial \Gamma}.\dot{\Gamma} = \int_{\Gamma} [\,\mathbf{q}\,]_{\Gamma}.\boldsymbol{\nu}\ \mathrm{d}S = -\int_{\Gamma} \mathcal{G}_{th}\phi\ \mathrm{d}S, \tag{28}$$

then the derivative of the total energy relatively with respect to the position of the interface determines the source of heat due to the irreversible process, intensity of which is governed by the internal ernergy release rate:

$$\mathcal{G}_{th} = -\frac{\partial \mathrm{E}}{\partial \Gamma}. \tag{29}$$

The entropy production is given by

$$\int_{\Omega} (\rho\dot{s} + \frac{\operatorname{div}\mathbf{q}}{T} - \mathbf{q}.\frac{\nabla T}{T^2})\ \mathrm{d}\Omega + \int_{\Gamma} (-m[\,s\,]_{\Gamma} - \boldsymbol{\nu}.[\,\frac{\mathbf{q}}{T}\,]_{\Gamma})\ \mathrm{d}S \geq 0. \tag{30}$$

Under the assumption of separability of the two dissipations, the term inside the volume is reduced to the conduction, and the term along the surface is then

$$D_\Gamma = \frac{\rho[\,w\,]_\Gamma - \sigma : [\,\varepsilon\,]_\Gamma}{T}\,\phi = \frac{\mathcal{G}_s}{T}\,\phi \geq 0, \tag{31}$$

where $\mathcal{G}_s$ is the release rate of free energy.

This quantity has an analogous form to the driving traction force acting on a surface of strain discontinuity introduced in [1] . The criteria which guide the evolution of the interface may be written as function of this quantity.

In a thermomechanical coupling, two different release rates must be distinguished. One defined in terms of variation of the total internal energy gives rise to the heat source associated with the moving surface ; the second one gives rise to the production of entropy.

In the case of isothermal evolution the total dissipation is given in terms of the derivative of the potential energy relatively to the position of the interface

$$\frac{\partial \mathcal{E}}{\partial \Gamma}.\dot{\Gamma} = -\int_\Gamma \mathcal{G}_s\phi\,\mathrm{d}S, \text{ or } \mathcal{G}_s(\,\mathbf{x}\,) = -\frac{\partial \mathcal{E}}{\partial \Gamma}(\,\mathbf{x}\,). \tag{32}$$

whith $\mathcal{G}_s = \rho[\,w\,]_\Gamma - \sigma : [\,\varepsilon\,]_\Gamma$.

In this case, there is only one energy release rate to characterize the propagation, it gives the sources of entropy production and the dissipation.

These relations can be generalized to the dynamical case, by replacing the internal energy of the system by its Hamiltonian, and can be extended to the case of running cracks, and to more general behaviour and structures [10].

4 Dissipation analysis in dynamical case.

Now, we take the inertia effects into account. Thus the two thermodynamics principles must be rewritten. The mass conservation leads to the continuity of the mass flux $m = \rho\phi$, where ρ denotes the mass density. The first law and the second law of thermodynamics give rise to local equations inside the volume and along the moving surface Γ :

$$\begin{aligned}\rho\dot{e}_i &= \sigma_i : \dot{\varepsilon} - divq, \text{ in } \Omega_i,\\ 0 &= m[\,\mathrm{e} + \frac{\mathbf{v}^2}{2}\,]_\Gamma - \boldsymbol{\nu}.[\,\sigma.\mathbf{v}\,]_\Gamma + \boldsymbol{\nu}.[\,\mathbf{q}\,]_\Gamma, \text{ on } \Gamma.\end{aligned}$$

Then taking the conservation of the momentum and the continuity of the displacement into account :

$$[\,\mathbf{u}\,]_\Gamma = 0, \quad [\,\sigma\,]_\Gamma.\boldsymbol{\nu} = m[\,\mathbf{v}\,]_\Gamma. \tag{33}$$

we obtain the heat power supply defined by the internal energy release rate $\mathcal{G}_{th}$ ($\bar{\sigma} = \frac{1}{2}(\sigma_1 + \sigma_2)$) :

$$-\boldsymbol{\nu}.[\,\mathbf{q}\,]_\Gamma = \mathcal{G}_{th}\phi, \quad \mathcal{G}_{th} = \rho[\,\mathrm{e}\,]_\Gamma - \boldsymbol{\nu}.\bar{\sigma}.[\,\nabla\mathbf{u}\,]_\Gamma.\boldsymbol{\nu}.$$

The Hamiltonian of the structure is the sum of the kinetic energy and the total internal energy, the potential energy is defined as above :

$$\mathcal{H} = \int_\Omega \frac{\mathbf{p}^2}{2\rho}\,\mathrm{d}\Omega + \mathcal{E} + \int_\Omega \rho s T\,\mathrm{d}\Omega \tag{34}$$

The momentum conservation is then defined by the set of equations

$$\frac{\partial \mathcal{H}}{\partial \mathbf{p}} \bullet \delta\mathbf{p} = \int_\Omega \mathbf{v}.\delta\mathbf{p}\,\mathrm{d}\Omega, \tag{35}$$

$$\frac{\partial \mathcal{H}}{\partial \mathbf{u}} \bullet \delta\mathbf{u} = -\frac{d}{dt}\int_\Omega \mathbf{p}.\delta\mathbf{u}\,\mathrm{d}\Omega, \tag{36}$$

where $\mathbf{p}$ is the momentum, these equations leads to the classical equation of motion. The first law of thermodynamics is rewritten as follows :

$$\frac{d\mathcal{H}}{dt} - \frac{\partial \mathcal{H}}{\partial \mathbf{T}^d}.\dot{\mathbf{T}}^d = \int_{\partial\Omega} -\mathbf{q}.\mathbf{n}\, \mathrm{d}S, \tag{37}$$

and taking into account of the momentum conservation, we have

$$\frac{\partial \mathcal{H}}{\partial \Gamma}.\dot{\Gamma} = \int_{\Gamma} [\,\mathbf{q}\,]_{\Gamma}.\boldsymbol{\nu}\, \mathrm{d}S = -\int_{\Gamma} \mathcal{G}_{th}\phi\, \mathrm{d}S, \tag{38}$$

the second law has the same form as previously. The interface is perfect at each time, under the assumption of separability of the two dissipations, the term inside the volume is reduced to the conduction, and the term along the surface is then : $D_{\Gamma} = \frac{\mathcal{G}_s}{T}\phi$ where $\mathcal{G}_s$ has also the form of a release rate of energy.

$$\mathcal{G}_s = \rho[\,w\,]_{\Gamma} - \boldsymbol{\nu}.\bar{\sigma}.[\,\nabla\mathbf{u}\,]_{\Gamma}.\boldsymbol{\nu}. \tag{39}$$

In a thermomechanical coupling, two different release rates must be distinguish, one defined in term of variation of the Hamiltonian give rise the heat source associated with the moving surface, the second one describes the production of entropy.

In the case of isothermal evolution, we can define another Hamiltonian

$$\mathcal{H} = \int_{\Omega} \frac{\mathbf{p}^2}{2\rho}\ \mathrm{d}\Omega + \mathcal{E}, \tag{40}$$

and the total dissipation is then given by :

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H} - \frac{\partial \mathcal{H}}{\partial \mathbf{T}^d}.\dot{\mathbf{T}}^d = \frac{\partial \mathcal{H}}{\partial \Gamma}.\dot{\Gamma} = -\int_{\Gamma} \mathcal{G}_{dyn}\phi\, \mathrm{d}S, \tag{41}$$

where $\mathcal{G}_{dyn} = \rho[\,w\,]_{\Gamma} - \boldsymbol{\nu}.\bar{\sigma}.[\,\nabla\mathbf{u}\,]_{\Gamma}.\boldsymbol{\nu}$.

5 A thermodynamical approach to contact wear

The system consists in two sliding contacting bodies Ω_1, Ω_2 separated by a contact interface Ω_3. We assume the properties of Ω_3 are known, and we attemp to characterized the behaviour and the evolution of the interface, taking into account modelization of wear phenomena. Such an interface Ω_3 must be

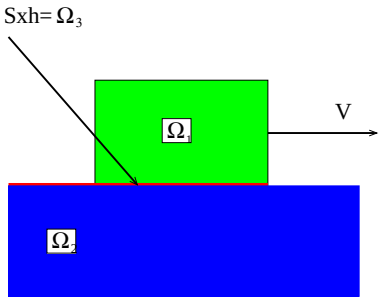

Fig. 1 Macroscopic description of contact.

considered at a macroscopic level as an homogeneous body obtained by some averaging process through the thickness H of Ω_3. This thickness is so small compared with the size of the contact zone and of the tribological system that the condition of homogeneity can be useful.

The behaviour of each zone is defined through a free energy w_i and a dissipation potential. As a result of wear, the boundary Γ_i moves. Along each front Γ_i, the normal $\boldsymbol{\nu}_i$ is oriented toward the sound

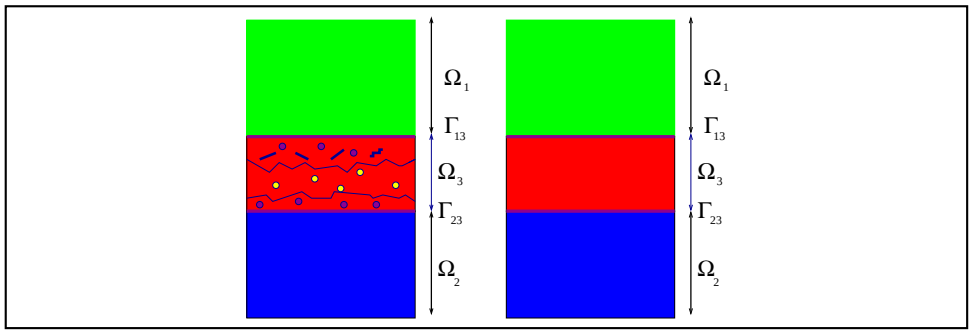

Fig. 2 The moving boundaries and the interface medium.

solid. We denote by $\phi_i \boldsymbol{\nu}_i$ the normal velocity of the surface Γ_i. Along these boundaries, mechanical quantities avec jump $[\,f\,]_{\Gamma_i} = f_i^+ - f_i^-$. By expressing the conservation laws in Ω_i and accross each Γ_i, we get a set of local equations for the characterisation of the rate quantities :

– mass conservation

$$\begin{cases} \text{on } \Gamma_i, & m_i = \rho_i \phi_i \boldsymbol{\nu}_i, \\ \text{in } \Omega_i, & \dot{\rho}_i + \operatorname{div} \rho_i \mathbf{v} = 0, \end{cases} \tag{42}$$

– momentum conservation

$$\begin{cases} \text{on } \Gamma_i, & \sigma.\boldsymbol{\nu}_i = 0, \\ \text{in } \Omega_i, & \operatorname{div} \sigma = 0, \end{cases} \tag{43}$$

– energy balance equation

$$\begin{cases} \text{on } \Gamma_i, & m_i[\,w + sT\,]_{\Gamma_i} - \boldsymbol{\nu}_i \sigma.[\,\mathbf{v}\,]_{\Gamma_i} + \boldsymbol{\nu}_i.[\,\mathbf{q}\,]_{\Gamma_i} = 0, \\ \text{in } \Omega_i, & \rho \dot{\mathrm{e}} = \sigma : \varepsilon(\mathbf{v}) - \operatorname{div} \mathbf{q}. \end{cases} \tag{44}$$

– continuity of the displacement along each Γ_i

$$[\,\mathbf{u}\,]_{\Gamma_i} = 0 \quad \Rightarrow [\,\mathbf{v}\,]_{\Gamma_i} + \phi_i [\,\nabla \mathbf{u}\,]_{\Gamma_i}.\boldsymbol{\nu}_i = 0. \tag{45}$$

6 The dissipation

The internal entropy production is positive and is decomposed in three contributions :

– the volume thermal conduction

$$D_{th} = -\mathbf{q}.\frac{\nabla T}{T^2}, \tag{46}$$

– the volume term due to intrinsic mechanical irreversibility

$$D_m = \frac{1}{T}\Big(\sigma : \operatorname{grad} \mathbf{v} - \rho(\dot{w} + s\dot{T})\Big), \tag{47}$$

– the surface term due to mechanical discontinuities

$$D_\Gamma = \frac{1}{T}(m_i[\,w\,]_{\Gamma_i} - \boldsymbol{\nu}_i.\sigma.[\,\mathbf{v}\,]_{\Gamma_i}). \tag{48}$$

We can notice that if the mass flux m_i is zero, the velocity ϕ_i is zero too. Then the velocity jump verifying Hadamard relation's is zero, and no dissipation along Γ_i occurs. The dissipation D_Γ is then a characteristic of loss of matter and so of the phenomenon of wear.

Description of the interface The interface Ω_3 is described by it's middle surface Γ with equation $S(X,t)=0$, normal vector $\mathbf{n}(X,t)$ and thickness $H(X,t)=2h(X,t)$.

The two boundaries Γ_{13},Γ_{23} are then defined by

$$x_1 = X + h(X,t)\mathbf{n}, \quad x_2 = X - h(X,t)\mathbf{n}. \tag{49}$$

Denoting ϕ the velocity of Γ, it is obvious that

$$\mathcal{D}_\phi(x_1) = \phi_1 \boldsymbol{\nu}_1, \quad \mathcal{D}_\phi(x_2) = \phi_2 \boldsymbol{\nu}_2. \tag{50}$$

The continuity of the displacement is then rewritten as

$$[\,\mathbf{u}(X \pm h(X,t)\mathbf{n},t)\,]_{\Gamma_i} = 0. \tag{51}$$

The dissipation by unit area of contact surface along Γ is then

$$D_m = D_{\Gamma_{13}} j_1 + D_{\Gamma_{23}} j_2 + \int_H d_m\; j(z)\; dz, \tag{52}$$

where $j_1 = j(h),\ j_2 = j(-h),\ j(z) = \det(\,\mathsf{I} - z\mathsf{b})$ and b is the curvature tensor.

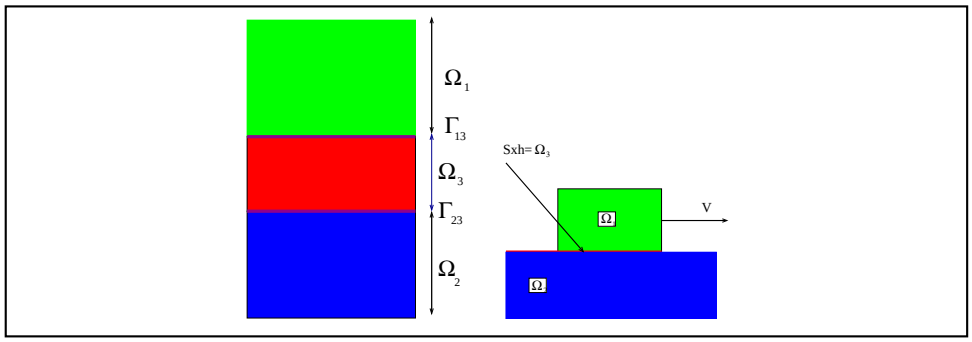

Fig. 3 The mesoscopic description.

Interpretation The contributions on the dissipation have different nature. The dissipation along the interface Γ_{13},Γ_{23} are characteristics of the loss of sound material. The last term is the dissipation due to the irreversibility inside the interface. For example, if Ω_3 is a viscous fluid, this term is due to the shear stresses, and consequently we have the possibility to describe some resistance to glide. The term D_3

$$D_3 = \int_h d_m\; j(z)\; dz = \int_H (\sigma : \dot{\varepsilon} - \dot{w})\; j(z)\; dz, \tag{53}$$

is interpreted in order to describe the friction associated with the relative motion of solids.

For a given mechanical behaviour of the interface, the resolution of the evolution equation must be completed by evolution laws in order to determine the propagation of the damage material. For example, we consider a Griffith's criterion :

$$\begin{cases} \mathcal{G}(X,t) < G_c, & \phi = 0, \\ \mathcal{G}(X,t) = G_c, & \phi \geq 0. \end{cases} \tag{54}$$

The main difficulty is to choose a good modelization for the behaviour in the interface and a procedure for its own identification.

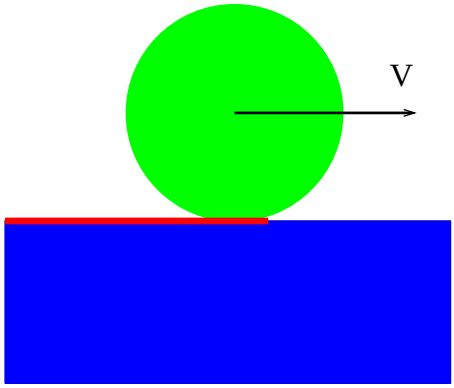

Fig. 4 A moving punch on an elastic half-space.

7 An example

Consider now an example of such a behaviour, to emphasize the possibility to describe the friction as well as the wear phenomena in the same approach. Consider a rigid punch Ω_1 moving on a half elastic plane. The interface is composed by a viscous fluid with particles in suspension, the global behaviour of this fluid is defined by a bulk modulus k and a viscous modulus η which are functions of the concentration of the particles. This particular case has been studied using of integral equations [4].

We consider that the shear is essentially due to viscosity and the elastic behaviour has only uniaxial effect in the direction $\mathbf{e}_y$. The motion inside Ω_3 is approximated by a linear profile for the velocities and displacement with respect to the normal coordinate of the middle surface S. Then $\dot{\varepsilon}$ is associated with the jump of the velocities.

Then the constitutive laws inside Ω_3 are given by :

$$\sigma_{xy} = m(c)(\dot{u}_x^1 - \dot{u}_x^2), \quad \sigma_{yy} = k(c)(u_y^1 - u_y^2). \tag{55}$$

These expressions are compatible with experimental observations. We are interested now on a steady-state solution, then

$$\dot{u}_x^1 - \dot{u}_x^2 = -V(\dot{u}_{x,x}^1 - \dot{u}_{x,x}^2). \tag{56}$$

The half plane has an elastic linear behaviour, the displacement on the interface Γ_{23} is given by solving the Galin's Equations :

$$co_1\ u_{x,x}(x) = co_2\ \sigma_{yy}(x) + Vp\frac{1}{\pi}\int_{-a}^{a}\frac{\sigma_{xy}(s)}{s-x}ds, \tag{57}$$

$$co_1\ u_{y,x}(x) = -co_2\ \sigma_{xy}(x) + Vp\frac{1}{\pi}\int_{-a}^{a}\frac{\sigma_{yy}(s)}{s-x}ds, \tag{58}$$

where the coefficients are :

$$co_1 = \frac{E}{2(1-\nu^2)}, \quad co_2 = \frac{(1-2\nu)}{2(1-\nu)}, \tag{59}$$

and E is the Young's modulus, ν is the Poisson's ratio, Vpf is the principal value of f in the sense of Cauchy.

The solution is founded by a method of perturbation, using an asymptotic expansion with respect to the concentration c of particles. Finally we get the following results :

- at the 0 order, the Hertz contact solution is recovered,
- at the first order, a dependance with the concentration is obtained. This depends upon the criterion of wear. For sake of simplicity a linear law is chosen, the velocity of progagation is given by $\phi = \lambda\sigma_{yy}^2$.

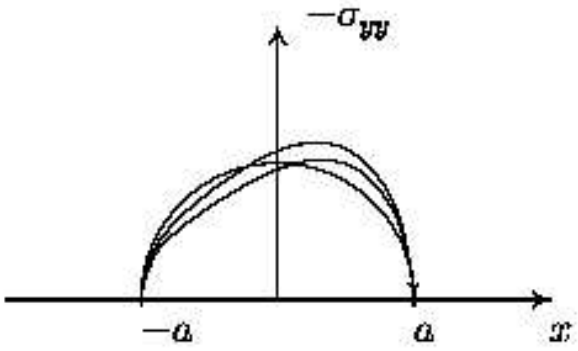

Fig. 5 Pressure in the contact area.

Due to the presence of a viscous fluid, the maximum pressure under the punch is not in the center of the contact area, as in the case of contact with friction, [2]. This example shows the pertinence of the modelization and the possibility of studying a problem of thin layer interface for describing the interaction between wear and friction.

Global approach of the interface With the same hypotheses, we take the average value of the free energy to define the global free energy of the interface per unit of contact area.

$$\psi_S(\mathbf{u}_1, \mathbf{u}_2, \alpha) = \frac{1}{\rho_S} \int_H \rho(x, z) w_3(\varepsilon(x + z\mathbf{n}), \alpha) dz, \tag{60}$$

where α denotes internal parameters, and ρ_S is the mass density defined by area unit too :

$$\rho_S = \int_H \rho(x, z) dz. \tag{61}$$

In the same way, we can define the potential of dissipation

$$\mathrm{D}_S(\mathbf{v}_1, \mathbf{v}_2, \dot{\alpha}) = \frac{1}{\rho_S} \int_H \rho(x, z) \mathrm{d}_3(\dot{\varepsilon}(x + z\mathbf{n}), \dot{\alpha}) dz. \tag{62}$$

As pointed out, the equilibrium along the interface is defined by the equations obtained by variations of the potential energy with respect to the fields $\mathbf{u}_1, \mathbf{u}_2$. Then we obtain :

$$\sigma.\mathbf{n}_i = \rho_S (\frac{\partial \psi_S}{\partial \mathbf{u}_i} + \frac{\partial \mathrm{D}_S}{\partial \mathbf{v}_i}). \tag{63}$$

This result suggests a more general study of the behaviour of the interface by asymptotic expansion of the displacement

$$\mathbf{u}_3 = \mathbf{u}_o + z\mathbf{u}_1 + z^2\mathbf{u}_2 + ... \tag{64}$$

with respect to the normal coordinate inside the interface Ω_3. Then specific functions for the constitutive behaviour should be deduced in the same manner.

8 Conclusion

We have developped a thermodynamical approach of contact wear based on moving interface description. In a such description, we emphasize that the wear is governed by local quantities. The dissipation is a function of a local energy release rate. Finally, in a global description we can consider the interface as a thin layer with specific properties. This allows us to take into account of modification of the properties of this interface with the loss of matter due to wear. The main difficulties are now to indentify with macroscopic experimental data such a complex behaviour depending on the degree of refinement of the description.

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